Data-Driven Multiagent Systems Consensus Tracking Using Model Free Adaptive Control

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***Abstract*— This paper investigates the data-driven consensus tracking problem for multiagent systems with both fixed communication topology and switching topology by utilizing a distributed model free adaptive control (MFAC) method. Here, agent's dynamics are described by unknown nonlinear systems and only a subset of followers can access the desired trajectory. The dynamical linearization technique is applied to each agent based on the pseudo partial derivative, and then, a distributed MFAC algorithm is proposed to ensure that all agents can track the desired trajectory. It is shown that the consensus error can be reduced for both time invariable and time varying desired trajectories. The main feature of this design is that consensus tracking can be achieved using only input–output data of each agent. The effectiveness of the proposed design is verified by simulation examples.**

***Index Terms*— Consensus tracking, data-driven design, model free adaptive control (MFAC), multiagent systems.**

I. Introduction

RECENTLY, distributed coordination of multiagent systems has been applied to many practical areas, such as satellite formation, autonomous underwater vehicles, automated highway systems, and mobile robots. As a result, an increasing number of researchers have been attracted to this area of study and considerable efforts have been focused on the problem of cooperative control for multiagent systems [1], [2]. Consensus is an important issue for multiagent systems. The task of the consensus problem is to find appropriate control strategies depending only on local knowledge such that multiagent systems can agree on certain quantities of interest.

A large number of effective control approaches have been reported to solve the multiagent consensus problem [3]–[13]. In [3] and [4], consensus problems are considered for dynamic agents with both fixed and switching topologies. In [5] and [6], the problem of consensus for multiagent systems with constrained information exchange is considered. In [7] and [8], the group consensus problem is addressed for multiagent systems using the leader-following approach. In [9], the consensus problem is discussed for agents described by double-integrator dynamics. In [10], a stationary consensus protocol is developed for multiagent systems with fixed topologies. In [11], the problem of cooperative output regulation is investigated for singular heterogeneous multiagent systems. The proposed cooperative controller depends on the interaction topologies and the plant parameters. In [12] and [13], consensus is studied for multiagent systems with stochastically switching graphs. It is shown that the almost sure consensus can be obtained in such kind of probabilistic conditions. In [14], the problem of finite-time consensus for multiagent systems on a fixed directed interaction graph is discussed. In [15], the consensus problem for a class of sampled data multiagent systems with packet losses is investigated. For the state-of-the-art approaches of consensus problems in multiagent systems, readers are referred to [16] and [17].

In the aforementioned related studies on consensus control, the dynamics of agents are usually assumed to be known. However, it is difficult to establish the exact system model for practical multiagent systems. Moreover, almost all the dynamics of the agents contain nonlinearities. Therefore, the study of consensus problems for unknown nonlinear multiagent systems is a challenging topic. Some studies have been conducted as follows. Neural networks (NNs) have excellent approximation abilities for nonlinear dynamics. Using this excellent property, adaptive NN consensus control for multiagent systems with unknown nonlinear dynamics has been studied in [18]–[21]. However, in these NNs-based adaptive controllers, some training processes and external testing signals are necessary for controller design. In addition, iterative learning control is an effective approach for formation or consensus tracking control for multiagent systems with unknown nonlinear uncertainties [22]–[25]. However, this method is established based on the assumption that coordination problems are required periodic executions or to be repeated.

Model free adaptive control (MFAC) is an effective approach for discrete-time nonlinear systems with unknown dynamics [26]–[30]. This approach does not need any identification of the unknown nonlinear model, and it proposes a novel dynamical linearization approach using the so-called pseudo partial derivative (PPD) of the nonlinear system. This linearization approach can establish the linear

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model for the unknown system along dynamic operation points. The PPD can be estimated on the basis of input–output (I/O) measurement data of the nonlinear system, and then, a model free control law can be designed.

This paper considers the consensus problem for unknown nonlinear multiagent systems by utilizing the MFAC approach. The dynamical linearization technique is applied to each agent based on the PPD, and then a distributed MFAC algorithm is proposed to ensure that all agents can track the desired trajectory. In comparison with the existing literatures, the main challenges and contributions of this paper are summarized as follows.

1) The consensus problem can be achieved using only real time measurement I/O data of the multiagent systems. It does not require any mathematical model or structural information for agents. This makes sense that the mathematical models of real-world multiagent systems are often difficult to obtain accurately, while the abovementioned previous works [3]–[15] consider the multiagent systems with known dynamics. Furthermore, since the model information of agents is not utilized in the controller design, the proposed consensus method has strong robustness for unmodeled uncertainties of agents.
2) To release the requirement of mathematical model, an alternative solution is adaptive consensus control approaches based on NNs as done in [18]–[21]. However, these approaches are only proposed for multiagent systems with affine nonlinear dynamics. To remove the restriction, the proposed distributed MFAC approach in this paper can solve consensus tracking problem for general nonaffine nonlinear multiagent systems. In addition, the proposed method does not need any training process or any external testing signals, which are usually necessary for adaptive NN consensus approaches [18]–[21].

The design in this paper is similar to the one in [27] and [28]. However, this paper considers the problem of distributed MFAC for multiagent systems. While the control law in [27] and [28] is designed for a single agent. This extension provides a new method to design data-driven consensus algorithms for multiagent systems by using MFAC and presents new problem for MFAC, where the control law is designed based on the neighbor-based tracking error.

The rest of this paper is organized as follows. Section II introduces the necessary preliminaries and then gives the problem formulation. Section III gives the distributed MFAC multiagent consensus algorithm and then analyzes the tracking performance. Section IV extends the design to multiagent systems with switching topologies. Three simulation examples are given in Section V. In Section VI, some conclusions are given and possible future work is also discussed.

II. Preliminaries and Problem Formulation

A. Preliminaries

The set of real numbers is expressed by R. $\|A\|$ is a matrix norm for a given matrix $A \in R^{n\times n}$. $\mathrm{diag}(\cdot)$ denotes the diagonal matrix and I is an identity matrix with an appropriate dimension. In multiagent systems, graph theory is an effective tool to model the interaction topologies. Next, we provide a brief introduction to directed graphs in algebraic graph theory. Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted directed graph, where $\mathscr{V} = \{1, 2, \ldots, N\}$ is the set of vertices, $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ is the set of edges, and $\mathscr{A}$ is the adjacency matrix. Let $\mathscr{V}$ also be the index set expressing agents. If the agent j can receive message from the agent i, then $(i, j) \in \mathscr{E}$ and j is called the child of i and i is the parent of j. $N_i = \{j \in \mathscr{V} | (j, i) \in \mathscr{E}\}$ denotes the neighborhood of the agent i. $\mathscr{A} = (a_{i,j}) \in R^{N\times N}$ denotes the weighted adjacency matrix of $\mathscr{G}$, where $a_{i,i} = 0, a_{i,j} = 1$ if $(j, i) \in \mathscr{E}$; otherwise $a_{i,j} = 0$. The Laplacian matrix of $\mathscr{G}$ is defined as $L = \mathscr{D} - \mathscr{A}$, where $\mathscr{D} = \mathrm{diag}(d_1^{\mathrm{in}}\ d_2^{\mathrm{in}} \cdots d_N^{\mathrm{in}})$ and $d_i^{\mathrm{in}} = \sum_{j=1}^{N} a_{i,j}$ is called the in-degree of vertex i. If there exists a path between any pair of two vertices, then the graph is said to be strongly connected.

B. Problem Formulation

In existing studies, the consensus problem is often considered for a group of agents with identical dynamics. However, heterogeneity is the intrinsic property for multiagent systems. Even if agents have same types and similar structures, it is impossible that they have identical parameters. Therefore, the problem of consensus for heterogeneous agents is more challenged. Consider a heterogeneous multiagent system with N agents. Their interaction topology is given by $\mathscr{G}$. Assume that the agent i is considered to have the following nonlinear dynamics:

$$y_i(k+1) = f_i(y_i(k), u_i(k)), \quad i = 1, 2, \ldots, N \tag{1}$$

where $y_i(k) \in R$ is the output, $u_i(k) \in R$ is the control input and $f_i(\cdot)$ is an unknown nonlinear function, respectively.

Give a desired consensus tracking trajectory $y_d(k)$. It is assume that $y_d(k)$ only can be accessed by a subset of agents. In addition, we assume that the desired trajectory is generated by a virtual leader and index it as vertex 0. Then, we can obtain a directed graph $\bar{\mathscr{G}} = (\mathscr{V} \cup \{0\}, \bar{\mathscr{E}}, \bar{\mathscr{A}})$ consisting of $N+1$ agents, where $\bar{\mathscr{E}}$ is the edge set and $\bar{\mathscr{A}}$ is the weighted adjacency matrix of $\bar{\mathscr{G}}$.

The following assumptions for nonlinear dynamics are given to facilitate our analysis.

Assumption 1: The partial derivative of nonlinear function $f_i(\cdot)$ with respect to $u_i(k)$ is continuous.

Assumption 2: The model (1) is generalized Lipschitz, that is, if $\Delta u_i(k) \neq 0$, $|\Delta y_i(k+1)| \leq b\,|\Delta u_i(k)|$ holds for any k, where $\Delta y_i(k+1) = y_i(k+1) - y_i(k)$, $\Delta u_i(k) = u_i(k) - u_i(k-1)$ and b is a positive constant.

Remark 1: The reasonability of the above two assumptions for practical nonlinear systems has been discussed in [24]. Hence, they are also acceptable for practical multiagent systems. Assumption 1 is a general condition for controller design. Assumption 2 means that the change rate of agent's output corresponding to the change rate of the agent's control input is bound. In view of system energy, if changes of the control input energy are finite, the output energy change rates cannot tend to infinity.

In the following theorem, we will demonstrate that, if the agent's dynamic stratifies Assumptions 1 and 2, then the nonlinear dynamic can be described as a dynamic linearization model [27]. Then, the distributed MFAC algorithm can be designed based on the model.

Theorem 1 [27]: Consider that the agent's dynamic (1) satisfies Assumptions 1 and 2. If $\Delta u_i(k) \neq 0$ holds, then system (1) can be described as a compact form dynamic linearization model

$$\Delta y_i(k+1) = \phi_i(k)\Delta u_i(k) \tag{2}$$

where $\phi_i(k)$ is a variable named pseudo-partial-derivative and it satisfies $|\phi_i(k)| \leq b$.

Define the following distributed measurement output:

$$\xi_i(k) = \sum_{j\in N_i} a_{i,j}(y_j(k) - y_i(k)) + d_i(y_d(k) - y_i(k)) \tag{3}$$

where $a_{i,j}$ is the (j,i)th entry in the adjacency matrix and N_i is the neighborhood set of the agent i. If the agent i can receive the desired trajectory, $d_i = 1$, i.e., $\{0, i\} \in \bar{\mathscr{E}}$ or there is an edge from the virtual leader to the agent i. Otherwise, $d_i = 0$.

Let $e_i(k) = y_d(k) - y_i(k)$ denote the tracking error. The objective of this paper is to find an appropriate control law only using the I/O data of the agents, such that the outputs of all agents can track the reference trajectory $y_d(k)$ when only some of agents can access the desired trajectory.

Assumption 3: The communication graph $\bar{\mathscr{G}}$ is a fixed strongly connected graph and at least one of the follower agents can access the leader's trajectory.

Remark 2: The communication condition in Assumption 3 is a necessary requirement for the solvability of the consensus tracking problem. If there is an isolated agent, which does not even know the control objective, it is impossible for that agent can follow the leader's reference trajectory.

Assumption 4: The PPD $\phi_i(k) > \varsigma$, $i = 1, 2, 3, \ldots, N$ (or $\phi_i(k) < -\varsigma$) holds for all k, where ς is an arbitrarily small positive constant. Without loss of generality, we assume $\phi_i(k) > \varsigma$ in this paper.

Remark 3: The above assumption indicates that the agent output does not decrease as the corresponding control input increases, which may be treated as a type of linear-like characteristic. This assumption implies that the sign of the control direction is known, or at least not changed. A similar assumption can also be found in model-based control approaches for the control direction. It is also a reasonable assumption for many practical multiagent systems, such as mobile robots and unmanned air vehicles.

III. Main Results

For the above consensus tracking objective, the following distributed MFAC algorithms is presented:

$$\hat{\phi}_i(k) = \hat{\phi}_i(k-1) + \frac{\eta\Delta u_i(k-1)}{\mu + |\Delta u_i(k-1)|^2} \times(\Delta y_i(k) - \hat{\phi}_i(k-1)\Delta u_i(k-1)) \tag{4}$$

$$\hat{\phi}_i(k) = \hat{\phi}_i(1), \quad \text{if } |\hat{\phi}_i(k)| \leq \varepsilon \quad \text{or} \quad \text{sign}(\hat{\phi}_i(k)) \neq \text{sign}(\hat{\phi}_i(1)) \tag{5}$$

$$u_i(k) = u_i(k-1) + \frac{\rho\hat{\phi}_i(k)}{\lambda + |\hat{\phi}_i(k)|^2}\xi_i(k). \tag{6}$$

where η, ρ are the step size,$\mu > 0$ and $\lambda > 0$ are weight factors. $\hat{\phi}_i(1)$ is the initial value of $\hat{\phi}_i(k)$ and $\hat{\phi}_i(k)$ is the estimated value of $\phi_i(k)$. ε is a small positive constant.

Remark 4: In the parameters estimation law (4), the data $\Delta y_i(k)$ is used to estimate $\hat{\phi}_i(k)$. The benefit of the scheme is that the convergence of parameters estimation law (4) can be guaranteed as shown in [27] and [28]. The reset algorithm (5) is given to guarantee that the parameter estimation algorithm can track time-varying parameter with stronger ability. In the control law (6), the distributed measurement output $\xi_i(k)$ for agent i is used to update the control input $u_i(k)$. Hence, the algorithm is a kind of distributed MFAC approach.

Remark 5: We can see that no agent model dynamics are included in the distributed MFAC scheme. The PPD parameters estimation algorithm and distributed control law are designed only depending on the measured I/O data of multiagent systems. Hence, it is a data-driven control approach for solving multiagent systems consensus tracking problem.

Remark 6: As mentioned in [27] and [28], the parameter estimation law (4) is to minimize the performance index $J(\phi_i(k)) = |\Delta y_i(k) - \hat{\phi}_i(k)\Delta u_i(k-1)|^2 + \mu|\hat{\phi}_i(k) - \hat{\phi}_i(k-1)|^2$. η is the step size and often selected as $0 < \eta < 1$. μ is a weighting factor and selected as $\mu > 0$. In the reset algorithm (5), ε is a small positive constant and often selected as 10^{-4} or 10^{-5}. Different from [27] and [28], here the control law (6) is to minimize the performance index $J(u_i(k)) = |\xi_i(k)|^2 + \lambda|u_i(k) - u_i(k-1)|^2$. λ is an important parameter for MFAC systems. From the existing theoretical analysis, it is shown that the stability of the MFAC system can be guaranteed by choosing a suitable λ. ρ is a controller parameter that can determine the stability condition. We will provide the condition in the following Theorems.

We first give the following lemma, which is used for the stability analysis.

Lemma 1 [31]: Let $W(k)$ denote a time varying irreducible substochastic matrix with positive diagonal entries and W denotes the set of all possible $W(k)$. Then, we have

$$\|W(P)W(P-1)\cdots W(1)\| \leq \beta$$

where $0 < \beta < 1$ and $W(k)$, $k = 1, 2, \ldots, P$, are P matrices arbitrarily selected from W.

Now, we give the stability of the proposed MFAC algorithm in the following Theorem.

Theorem 2: Consider that the multiagent system (1) satisfies Assumptions 1, 2, 4 and the communication graph satisfies Assumption 3. Let the proposed MFAC algorithm (4)–(6) be used. Assume that the desired reference trajectory $y_d(k)$ is time invariable, i.e., $y_d(k) = \text{const}$. If we select ρ such that it satisfies the condition

$$\rho < \frac{1}{\max_{i=1,\ldots,N}\sum_{j=1}^{N} a_{i,j} + d_i}$$

then there exists a $\lambda_{\min} > 0$ and $\lambda > \lambda_{\min}$ such that the tracking error $e_i(k)$ converges to 0 as $k \to \infty$ for all $i = 1, 2, \ldots, N$.

Proof: The proof comprises three parts as follows.

Part (i) (Boundedness of the Estimation Value $\hat{\phi}_i(k)$): Define $\tilde{\phi}_i(k) = \hat{\phi}_i(k) - \phi_i(k)$. From Theorem 1 and parameter estimation algorithm (4), we have

$$\tilde{\phi}_i(k) = \left(1 - \frac{\eta \Delta u_i(k-1)^2}{\mu + |\Delta u_i(k-1)|^2}\right) \times \tilde{\phi}_i(k-1) + \phi_i(k-1) - \phi_i(k). \tag{7}$$

We can obtain from (7) that

$$|\tilde{\phi}_i(k)| \leq \left|\left(1 - \frac{\eta \Delta u_i(k-1)^2}{\mu + |\Delta u_i(k-1)|^2}\right)\right| |\tilde{\phi}_i(k-1)| + |\phi_i(k-1) - \phi_i(k)|. \tag{8}$$

Since $|\Delta u_i(k)| \neq 0$, by selecting η, μ properly, such as $0 < \eta \leq 1$ and $\mu \geq 0$, there exists a constant q_1 such that

$$0 < \left|\left(1 - \frac{\eta \Delta u_i(k-1)^2}{\mu + |\Delta u_i(k-1)|^2}\right)\right| \leq q_1 < 1 \tag{9}$$

holds. Since $|\phi_i(k)| \leq b$, considering Assumption 4, we can obtain $|\phi_i(k-1) - \phi_i(k)| \leq b$.

From (8) and (9), we have

$$\begin{aligned} |\tilde{\phi}_i(k)| &\leq q_1 |\tilde{\phi}_i(k-1)| + b \\ &\leq \cdots \leq q_1^{k-1} |\tilde{\phi}_i(1)| + \frac{b\left(1 - q_1^{k-1}\right)}{1 - q_1} \end{aligned} \tag{10}$$

which indicates that $\tilde{\phi}_i(k)$ is bounded. Hence, $\hat{\phi}_i(k)$ is also bounded since $\phi_i(k)$ is bounded.

Part (ii) (The Convergence of Tracking Error): From (3), $\xi_i(k)$ can be rewritten in terms of tracking errors as follows:

$$\xi_i(k) = \sum_{j \in N_i} a_{i,j}(e_i(k) - e_j(k)) + d_i e_i(k). \tag{11}$$

Define the following column stack vectors:

$$\begin{aligned} \mathbf{y}(k) &= [y_1(k) \quad y_2(k) \quad \cdots \quad y_N(k)]^T \\ \mathbf{e}(k) &= [e_1(k) \quad e_2(k) \quad \cdots \quad e_N(k)]^T \\ \xi(k) &= [\xi_1(k) \quad \xi_2(k) \quad \cdots \quad \xi_N(k)]^T \\ \mathbf{u}(k) &= [u_1(k) \quad u_2(k) \quad \cdots \quad u_N(k)]^T. \end{aligned}$$

In this case, (11) can be described in a compact form as

$$\xi(k) = (L + D)\mathbf{e}(k) \tag{12}$$

where $D = \text{diag}(d_1, d_2, \ldots, d_N)$.

By noting the definition in (12), we can rewritten the distributed MFAC algorithm (6) as

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \rho H_1(k)(L + D)\mathbf{e}(k) \tag{13}$$

where

$$H_1(k) = \text{diag}\left(\frac{\hat{\phi}_1(k)}{\lambda + |\hat{\phi}_1(k)|^2} \frac{\hat{\phi}_2(k)}{\lambda + |\hat{\phi}_2(k)|^2} \cdots \frac{\hat{\phi}_N(k)}{\lambda + |\hat{\phi}_N(k)|^2}\right).$$

Following a similar way, we can also describe (2) as:

$$\mathbf{y}(k+1) = \mathbf{y}(k) + H_\phi(k)\Delta\mathbf{u}(k) \tag{14}$$

where

$$\begin{aligned} \Delta\mathbf{u}(k) &= \mathbf{u}(k) - \mathbf{u}(k-1) \\ H_\phi(k) &= \text{diag}(\phi_1(k) \ \phi_2(k) \ \cdots \ \phi_N(k)). \end{aligned}$$

We can substitute (13) into (14) to get

$$\begin{aligned} \mathbf{e}(k+1) &= \mathbf{e}(k) - \rho H_\phi(k) H_1(k)(L + D)\mathbf{e}(k) \\ &= (I - \rho\Sigma(k)(L + D))\mathbf{e}(k). \end{aligned} \tag{15}$$

where $\Sigma\ (k) = H_\phi(k)H_1(k) = \text{diag}(\vartheta_1(k)\ \vartheta_2(k)\ \cdots\ \vartheta_N(k))$ and

$$\vartheta_i(k) = \frac{\phi_i(k)\hat{\phi}_i(k)}{\lambda + |\hat{\phi}_i(k)|^2}, \quad i = 1, 2, \ldots N.$$

Denote $\Theta\ (k) = \Sigma\ (k)(L + D)$. From (15), we can obtain that if $\|I - \rho\Theta\ (k)\| < 1$ for all k, then $\lim_{k\to\infty} \|\mathbf{e}(k+1)\| = 0$.

Part (iii) (Deriving Convergence Condition at the Agent Level): In the following, we derive the condition at the agent level.

Since $\phi_i(k)$ and $\hat{\phi}_i(k)$ are bounded for all $i = 1, 2, \ldots, N$, there exists a bounded constant $\lambda_{\min} > 0$ such that if $\lambda > \lambda_{\min}$, the following inequality holds:

$$0 < \vartheta_i(k) = \frac{\phi_i(k)\hat{\phi}_i(k)}{\lambda + |\hat{\phi}_i(k)|^2} \leq \frac{b\hat{\phi}_i(k)}{2\sqrt{\lambda}|\hat{\phi}_i(k)|} < \frac{b}{2\sqrt{\lambda_{\min}}} < 1.$$

On the other hand, since the communication graph is strongly connected, $I - \rho\Theta\ (k)$ must be an irreducible matrix. If we select ρ such that it satisfies the condition

$$\rho < \frac{1}{\max_{i=1,\ldots,N} \sum_{j=1}^N a_{i,j} + d_i}$$

then ρ is less than the reciprocal of greatest diagonal entry of $L + D$. Notice that $0 < \vartheta_i(k) < 1$ for all $i = 1, 2, \ldots, N$, we can obtain at least one row sum of $I - \rho\Theta\ (k)$ is strictly less than one. Hence, $I - \rho\Theta\ (k)$ is an irreducible substochastic matrix with positive diagonal entries.

We can obtain from (15) that

$$\begin{aligned} &\mathbf{e}(k+1) \\ &\leq \|I - \rho\Theta(k)\|\|\mathbf{e}(k)\| \\ &\leq \|I - \rho\Theta(k)\|\|I - \rho\Theta(k-1)\|\|\mathbf{e}(k-1)\| \\ &\leq \|I - \rho\Theta(k)\|\|I - \rho\Theta(k-1)\| \cdots \|I - \rho\Theta(1)\|\|\mathbf{e}(1)\|. \end{aligned} \tag{16}$$

By applying Lemma 1, group every P matrices product together in (16), we can obtain

$$\|\mathbf{e}(k+1)\| \leq \beta^{\lfloor \frac{k}{P} \rfloor}\|\mathbf{e}(1)\|$$

where $\lfloor\cdot\rfloor$ stands for the floor function. Hence, we can conclude that $\lim_{k\to\infty} \|\mathbf{e}(k+1)\| = 0$.

This completes the proof.

Remark 7: We can see that the condition in Theorem 2 depends on communication graph G because $a_{i,j}, d_i$ are the elements of L and D. Therefore, Theorem 2 reveals the relationship between convergence property and communication topology. Under such a condition, the problem of consensus tracking can be handled by using the distributed MFAC scheme.

Next, we consider the time varying desired trajectory. Define the following column stack vectors:

$$\mathbf{y}_d(k) = [y_d(k) \quad y_d(k) \quad \cdots \quad y_d(k)]^T_{1\times N}.$$

Denote $\Delta\mathbf{y}_d(k) = \mathbf{y}_d(k+1) - \mathbf{y}_d(k)$, since $y_d(k)$ is bound, we have $\|\Delta\mathbf{y}_d(k)\| < b_y$, where b_y is a positive constant. The result of this case is summarized in the following Theorem.

Theorem 3: Consider that the multiagent system (1) satisfies Assumptions 1, 2, and 4 and communication graph satisfies Assumption 3. Let the proposed MFAC algorithm (4)–(6) be used. Assume that the desired trajectory $y_d(k)$ is time varying. If we select ρ such that it satisfies the condition

$$\rho < \frac{1}{\max_{i=1,\ldots,N}\sum_{j=1}^{N} a_{i,j} + d_i}$$

then there exists a $\lambda_{\min} > 0$ and $\lambda > \lambda_{\min}$ such that the tracking errors are uniformly ultimately bounded and the ultimate bound depends on the variation of desired trajectory.

Proof: In this case, the tracking error equation in (15) becomes

$$\mathbf{e}(k+1) = (I - \rho\Theta(k))\mathbf{e}(k) + \Delta\mathbf{y}_d(k). \tag{17}$$

We can obtain from (17) that

$$\begin{aligned}
&\|\mathbf{e}(k+1)\| \\
&\quad\le \|I-\rho\Theta(k)\|\|\mathbf{e}(k)\| + \|\Delta\mathbf{y}_d(k)\| \\
&\quad\le \|I-\rho\Theta(k)\|\|I-\rho\Theta(k-1)\|\|\mathbf{e}(k-1)\| \\
&\qquad + \|I-\rho\Theta(k)\|\|\Delta\mathbf{y}_d(k-1)\| + \|\Delta\mathbf{y}_d(k)\| \\
&\quad\le \|I-\rho\Theta(k)\|\|I-\rho\Theta(k-1)\|\cdots\|I-\rho\Theta(1)\|\|\mathbf{e}(1)\| \\
&\qquad + \|I-\rho\Theta(k)\|\|I-\rho\Theta(k-1)\|\cdots\|I-\rho\Theta(2)\|b_y \\
&\qquad + \ldots + \|I-\rho\Theta(k)\|b_y + b_y.
\end{aligned} \tag{18}$$

By applying Lemma 1, group every P matrices product together in (18), we can obtain

$$\begin{aligned}
&\|\mathbf{e}(k+1)\| \\
&\quad\le \beta^{\lfloor \frac{k}{P}\rfloor}\|\mathbf{e}(1)\| \\
&\qquad + \left(\beta^{\lfloor \frac{k-1}{P}\rfloor} + \beta^{\lfloor \frac{k-2}{P}\rfloor} + \cdots + \beta^{\lfloor \frac{1}{P}\rfloor} + \beta^{\lfloor \frac{0}{P}\rfloor}\right) b_y.
\end{aligned} \tag{19}$$

Donate $\alpha\ (k) = \beta^{\lfloor (kP/P)\rfloor} + \cdots + \beta^{\lfloor ((k+1)P-1/P)\rfloor}$, using the property of the floor function, we can obtain

$$\alpha\ (k) = (P-1)\beta^k.$$

Note that

$$\begin{aligned}
&\lim_{k\to\infty}\left(\beta^{\lfloor \frac{k-1}{P}\rfloor} + \beta^{\lfloor \frac{k-2}{P}\rfloor} + \cdots + \beta^{\lfloor \frac{0}{P}\rfloor}\right) \\
&\quad = \lim_{k\to\infty}\left(\beta^{\lfloor \frac{(k+1)P-1}{P}\rfloor} + \beta^{\lfloor \frac{(k+1)P}{P}\rfloor} + \cdots + \beta^{\lfloor \frac{0}{P}\rfloor}\right) \\
&\quad = \lim_{k\to\infty}(\alpha(k) + \alpha(k-1) + \cdots + \alpha(0)) \\
&\quad = (P-1)\lim_{k\to\infty}(\beta^k + \beta^{k-1} + \cdots + \beta^0) \\
&\quad = \frac{(P-1)}{1-\beta}
\end{aligned} \tag{20}$$

from (19) and (20), we can obtain

$$\lim_{k\to\infty}\|\mathbf{e}(k+1)\| = \frac{(P-1)}{1-\beta}b_y. \tag{21}$$

Hence, the tracking error is uniformly ultimately bounded and the bound depends on b_y.

This completes the proof.

Remark 8: It can be seen that, if the desired trajectory is time varying, the tracking error converges to a bound depending on the bound of $\Delta\mathbf{y}_d(k)$. If $b_y = 0$, then $\lim_{k\to\infty}\|\mathbf{e}(k)\| = 0$. In many practical tasks, the desired outputs are often slowly varying trajectories. This means that the bound of $\Delta\mathbf{y}_d(k)$ is small and the tracking error will converge to a small bound.

IV. Extension to Switching Topology

In this section, the proposed design is extended to the multiagent system with switching topology.

Let $\mathscr{G}(k)$ denote a time-varying graph depending on k. The adjacency matrix associated with $\mathscr{G}(k)$ is denoted by $\mathscr{A}(k) = (a_{i,j}(k)) \in R^{N\times N}$ and the Laplacian of $\mathscr{G}(k)$ is denoted by $L(k)$. The neighborhood of the ith agent is denoted by the set $N_i(k)$. To describe the switching topology, let $\mathscr{G}_\sigma = \{\mathscr{G}_1, \mathscr{G}_2, \ldots \mathscr{G}_M\}$ denote the set of all directed graphs for the agents, where $M \in Z^+$ denotes the total number of possible interaction graphs. The Laplacian of $\mathscr{G}_l$ is denoted by L_l for $l = 1, 2, \ldots, M$. We also view the desired trajectory as a virtual leader, and index it by the vertex 0 in the graph representation. In this case, the complete information flow of the whole interaction topology can be described as $\bar{\mathscr{G}}(k)$. In addition, $\bar{\mathscr{G}_\sigma} = \{\bar{\mathscr{G}_1}, \bar{\mathscr{G}_2}, \ldots \bar{\mathscr{G}_M}\}$ denotes the set of the finite possible interaction graphs for $\bar{\mathscr{G}}(k)$.

In this case, the definition in (3) becomes

$$\begin{aligned}
\xi_i(k) = &\sum_{j\in N_i(k)} a_{i,j}(k)(y_j(k) - y_i(k)) \\
&+ d_i(k)(y_d(k) - y_i(k)).
\end{aligned} \tag{22}$$

Assumption 5: The communication graph $\bar{\mathscr{G}_l}$ is a fixed strongly connected graph and at least one of the follower agents can access the leader's trajectory for all $l = 1, 2, \ldots M$.

Denote $D(k) = \text{diag}(d_1(k), d_2(k), \ldots, d_N(k))$ and D_l is the matrix defined in Section III for the interaction graph $\bar{\mathscr{G}_l}$. Then, we can give the following result.

Theorem 4: Consider that the multiagent system (1) satisfies Assumptions 1, 2, 4 and communication graph satisfies Assumption 5. Let the proposed MFAC algorithm (4)–(6) be used. Assume that the desired trajectory $y_d(k)$ is time invariable, that is $y_d(k) = \text{const}$. If we select ρ such that it satisfies the condition

$$\rho < \frac{1}{\max_{i=1,2,\ldots N,l=1,2,M}\sum_{j=1}^{N} a^l_{i,j} + d^l_i}$$

where $a^l_{i,j}$ is the element of L_l and d^l_i is the element of D_l. Then there exists a $\lambda_{\min} > 0$ and $\lambda > \lambda_{\min}$ such that the tracking errors $e_i(k)$ converge to 0 as $k \to \infty$ for all $i = 1, 2, \ldots, N$.

Proof: The compact form of (22) can be described as

$$\xi(k) = (L(k) + D(k))\mathbf{e}(k). \tag{23}$$

Then, the tracking error in (15) becomes

$$\mathbf{e}(k+1) = (I - \rho\Sigma(k)(L(k) + D(k)))\mathbf{e}(k). \quad (24)$$

Denote $\Lambda(k) = \Sigma(k)(L(k)+D(k))$. From (24), we can obtain that if $\|I - \rho\Lambda(k)\| < 1$ for all k, then $\lim_{k\to\infty} \|\mathbf{e}(k+1)\| = 0$.

Since all the possible interaction graphs are strongly connected, $I - \rho\Lambda(k)$ must be an irreducible matrix. Note that the set $\{L_1 + D_1, L_2 + D_2, \ldots, L_M + D_M\}$ contains all the possible matrices of $L(k) + D(k)$. If we choose ρ such that it satisfies the condition

$$\rho < \frac{1}{\max_{i=1,2,\ldots N, l=1,2,M} \sum_{j=1}^{N} a_{i,j}^{l} + d_i^l}$$

then ρ is less than the reciprocal of greatest diagonal entry of all the possible matrices $L(k) + D(k)$. Therefore, $I - \rho\Lambda(k)$ is an irreducible substochastic matrix with positive diagonal entries. Similar to the proof of Theorem 1, there exists a $\lambda_{\min} > 0$ and $\lambda > \lambda_{\min}$ such that the tracking errors $e_i(k)$ converge to 0 as $k \to \infty$ for all $i = 1, 2, \ldots, N$.

This completes the proof.

Remark 9: From Theorem 4, we can observe that, it is possible to derive consensus tracking for multiagent systems with a time invariable desired trajectory, although the interaction graph between agents is time varying. Similarly, the tracking results for a time varying desired trajectory can also be given by following the result of Theorem 3, which has been omitted here.

Remark 10: From Theorems 2 and 3, we can conclude that the distributed MFAC has several attractive properties. First, distributed MFAC uses only the real time measurement I/O data of the each agent. No mathematical model of the agent's dynamic is needed, which implies that we can develop a generic consensus control algorithm for a certain class of multiagent systems. Second, distributed MFAC does not require any training process and external testing signals, which are necessary for NN-based nonlinear adaptive consensus tracking control approaches. Third, since the agent's dynamic model information does not include in the distributed MFAC scheme, and then it has strong robustness for traditional unmodeled dynamics comparing with the other model based consensus control methods. Finally, the distributed MFAC is simple and easily implemented with small computational cost.

Remark 11: In this paper, the distributed MFAC is considered for multiagent systems with single input and single output nonlinear dynamics. However, many practical systems contain multiple inputs and multiple outputs. Due to the complexity of MIMO nonlinear systems, the proposed design in this paper cannot be directly extended to MIMO systems. One of the main difficulties in extending the proposed design to MIMO systems is input coupling. Hence, the distributed MFAC design for MIMO multiagent systems will be discussed in our future work.

V. Simulation Example

Example 1: In this example, we perform numerical simulations to illustrate the proposed consensus tracking results for fixed communication topology. Consider a network comprising four follower agents and the models are governed by

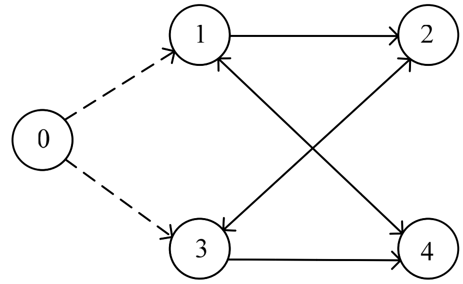

Fig. 1. Communication topology among agents of Example 1.

$$\text{Agent 1: } y_1(k+1) = \frac{y_1(k)u_1(k)}{1+y_1^2(k)} + u_1(k)$$
$$\text{Agent 2: } y_2(k+1) = \frac{y_2(k)u_2(k)}{1+y_2^3(k)} + 0.5u_2(k)$$
$$\text{Agent 3: } y_3(k+1) = \frac{y_3(k)u_3(k)}{1+y_3^2(k)} + 0.9u_3(k)$$
$$\text{Agent 4: } y_4(k+1) = \frac{y_4(k)u_4(k)}{1+y_4^5(k)} + 0.8u_4(k).$$

It can be seen that the considered agents are heterogeneous since their dynamics are different from each other. In this example, the dynamics are all unknown. They are given here only to generate the I/O data for the multiagent system and no model information is used in the distributed MFAC algorithm.

As the illustration shows in Fig. 1, the virtual leader is denoted as vertex 0. It can be observe that only agent 1 and 3 can receive information from the leader. Although agents 2 and 4 cannot receive the information of the leader, the communication graph is strongly connected. Assume that the information exchange among agents is directed and fixed. Then, the Laplacian matrix of the graph is given as follows:

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

and $D = \text{diag}(1\ 0\ 1\ 0)$. The reciprocal of greatest diagonal entry of $L + D$ is 0.5. If we select the controller parameters as $\rho = 0.3$, then the convergence condition in Theorem 2 and Theorem 3 is satisfied for all $i = 1, 2, 3, 4$. We consider the following two different desired trajectories.

A. Time Invariable Desired Trajectory

The desired trajectory is given as

$$y_d(k) = \begin{cases} 2, & 0 < k < 200 \\ 0.5, & 200 \le k < 400. \end{cases}$$

Initial conditions are chosen as $u_i(1) = 0$, $\hat{\phi}_i(1) = 2$ for all agents and $y_1(1) = 0.5$, $y_2(1) = 2.5$, $y_3(1) = 3.5$, $y_4(1) = 4$ in this simulation. The controller parameters are selected as $\eta = 1$, $\mu = \lambda = 0.5$, $\varepsilon = 10^{-5}$. Figs. 2–4 give the consensus

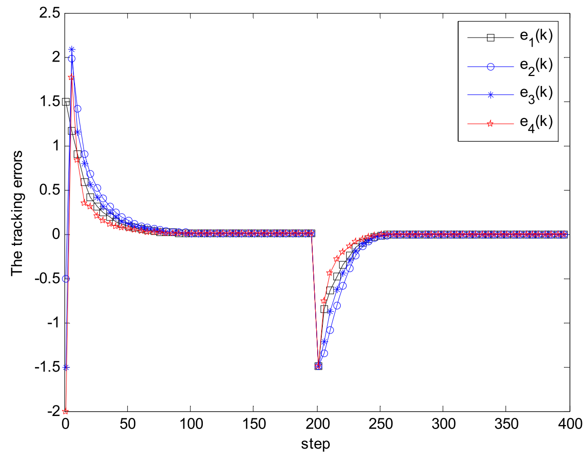

Fig. 2. Consensus tracking errors for time invariable desired trajectory (Example 1).

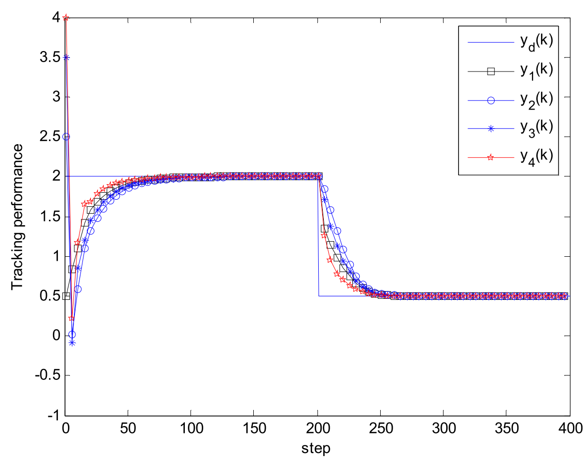

Fig. 3. Tracking performance for time invariable desired trajectory (Example 1).

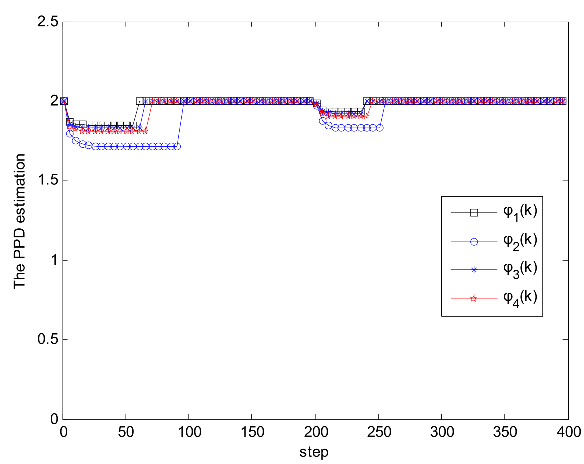

Fig. 4. PPD estimation for time invariable desired trajectory (Example 1).

tracking errors, system outputs, and the PPD estimation of all agents respectively. It can be observe that followers' outputs have large deviations from the desired one at the beginning time. However, the tracking errors can be reduced gradually

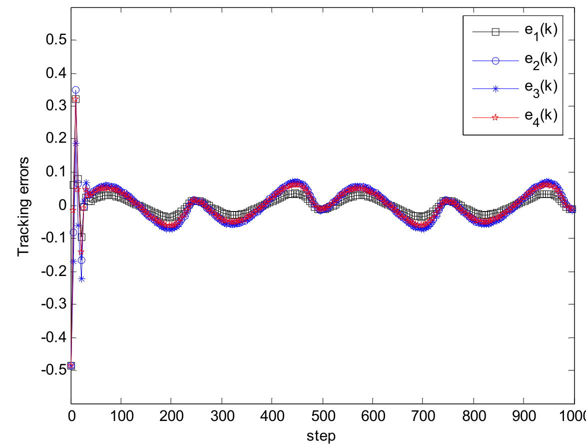

Fig. 5. Consensus tracking errors for time varying desired trajectory (Example 1).

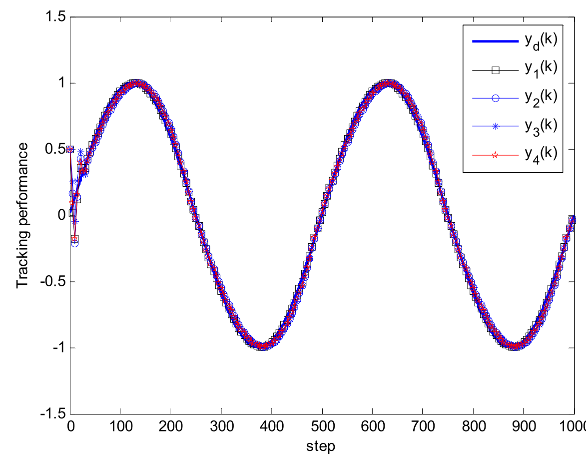

Fig. 6. Tracking performance for time varying desired trajectory (Example 1).

and the consensus tracking is achieved after time instant $k = 50$. Even though the desired trajectory is varied at $k = 200$, the consensus tracking objective can be also achieved after $k = 250$.

B. Time Varying Desired Trajectory

In this case, we consider the following desired trajectory:

$$y_d(k) = \sin\left(\frac{\pi k}{250}\right), \quad 0 \le k \le 1000.$$

Initial conditions are chosen as $u_i(1) = 0$, $y_1(1) = 0.5$, $\hat{\phi}_i(1) = 2$ for all agents and controller parameters are selected as $\eta = \mu = 1$, $\lambda = 0.1$, $\varepsilon = 10^{-5}$. Figs. 5–7 give the consensus tracking errors, system outputs and the PPD estimation of all agents, respectively. It can be found that the tracking errors are also gradually reduced by the MFAC controllers. However, the tracking errors cannot converge to 0, but they converge to a small bound. Obviously, this verifies that the consensus tracking result in Theorem 3 can be accomplished in the face of time varying desired trajectory.

In addition, to demonstrate the performance of the proposed algorithm, we perform a simulation by using consensus

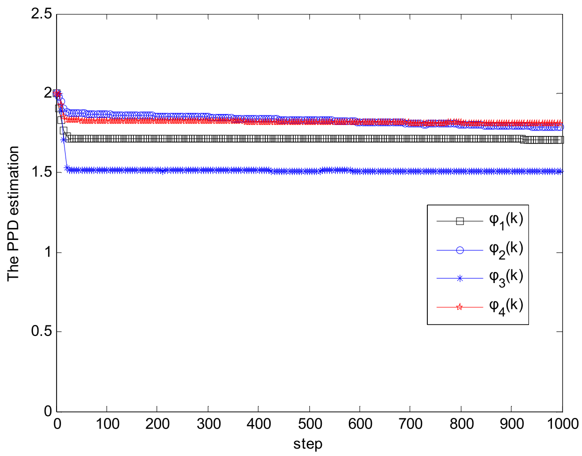

Fig. 7. PPD estimation for time varying desired trajectory (Example 1).

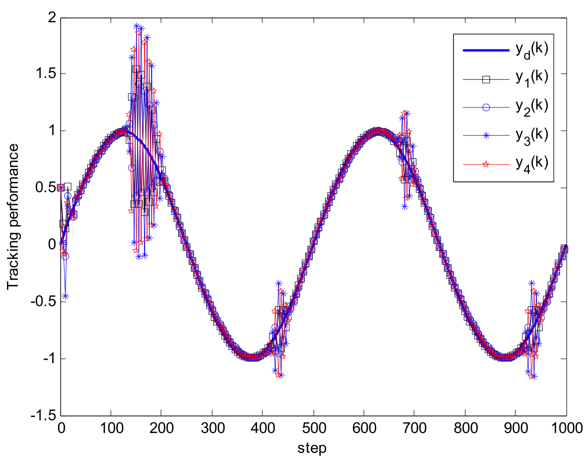

Fig. 8. Tracking performance using P-type distributed control algorithm (Example 1).

control algorithm with a fixed gain. Consider the P-type distributed control algorithm $u_i(k) = u_i(k-1) + K_i \xi_i(k)$. In the simulation, the same initial conditions are applied and $K_i = 0.8$ is selected for all agents. Fig. 8 shows the simulation result of tracking performance. From the comparison between Figs. 6 and 8, we can obviously see the effective of the distributed MFAC algorithm.

Example 2: In this example, we perform a simulation for multiagent systems with switching topologies. Consider the same multiagent system in example 1, here the communication graphs are given to switch over three states, i.e., $\mathscr{G}_\sigma = \{\mathscr{G}_1, \mathscr{G}_2, \mathscr{G}_3\}$, where the directed graphs are given in Fig. 9. To simulate the switching of the network topologies, we produce a stochastic switching signal $\sigma\ (k)$ as a function of k, which takes the values of 1, 2, and 3 randomly. Fig. 10 shows the switching signal $\sigma\ (k)$ in this simulation.

We also adopt 0–1 weighting in the adjacency matrix. Select the controller parameters as $\rho = 0.3$, it can be seen that the condition in Theorem 4 is satisfied for all $i = 1, 2, 3, 4$ and $l = 1, 2, 3$. Here, we consider the time invariable desired trajectory in example 1 and the same initial conditions are utilized for all agents. Figs. 11 and 12 show consensus tracking errors and the system outputs of all agents respectively. It can

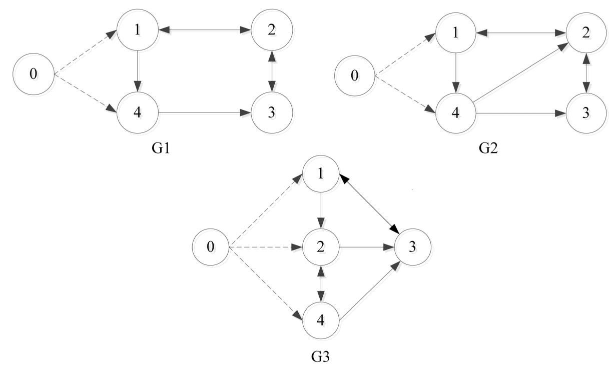

Fig. 9. Communication topology among agents of Example 2.

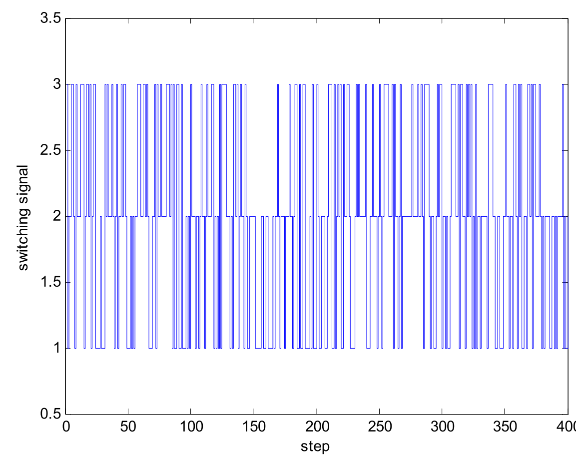

Fig. 10. Switching signal among there different communication graphs (Example 2).

be observed from these figures that the agents are enabled to track the desired trajectory after some time steps. This verifies the consensus tracking result in Theorem 4. Furthermore, from the comparison between tracking errors in Figs. 2 and 11, we can see that the curve in Fig. 11 is not smooth, which is caused by the switching topology.

Example 3: In this example, we perform a simulation for realistic mechanical systems, which comprise six permanent magnet dc linear motors. The dynamic of every motor is described as follows [32], [33]:

$$\begin{cases} \dot{x}(t) = v(t) \\ \dot{v}(t) = \dfrac{u(t) - f_{\text{friction}}(t) - f_{\text{ripple}}(t)}{m} \end{cases} \tag{25}$$

where $f_{\text{friction}}(t)$ is the friction force (N), $f_{\text{ripple}}(t)$ is the ripple force (N), $u(t)$ is the developed force (N), m is the combined mass of translator and load, $x(t)$ is position (m) and $v\ (t)$ is the speed (m/s), t is continuous time (s).

The friction and ripple forces are assumed to be modeled as follows:

$$f_{\text{friction}}(t) = (f_c + (f_s - f_c)e^{-(\dot{x}/\dot{x}_\delta)^\delta} + f_v \dot{x})sgn\dot{x}$$
$$f_{\text{ripple}}(t) = b_1 \sin\ (\omega_0 x(t))$$

where f_s is the level of static friction, f_c is the minimum level of Coulomb friction, x_δ and f_v are lubricant and load

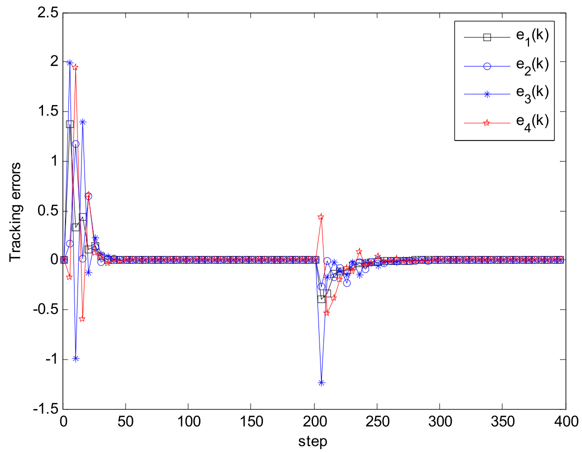

Fig. 11. Consensus tracking errors of Example 2.

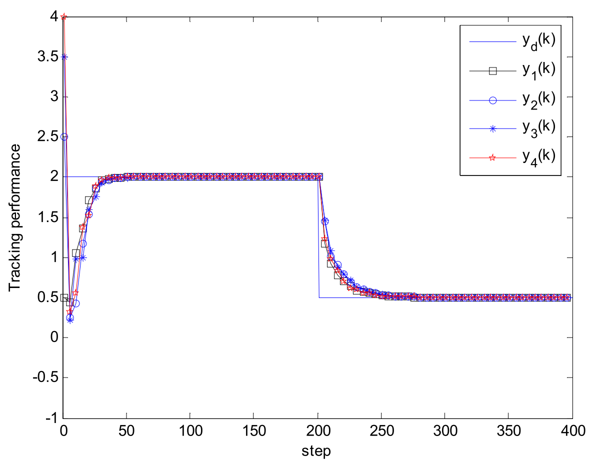

Fig. 12. Tracking performance of Example 2.

parameters. δ is an additional empirical parameter. In the simulation, these parameters are selected as: $m = 0.59$ kg, $x_\delta = 0.1$, $\delta = 1$, $f_c = 10$ N, $f_s = 20$ N, $f_v = 10$ N $\cdot$ s $\cdot$ m^{-1}, $b_1 = 8.5$ N, $\omega_0 = 314$ s^{-1}.

Denote $x_1(t) = x(t), x_2(t) = v\ (t)$, we can describe (25) as the following nonlinear dynamic:

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -\dfrac{f_{\text{friction}}(t) + f_{\text{ripple}}(t)}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} u(t) \\ y(t) = x_2(t). \end{cases}$$

The dynamics are also unknown. They are given here only to generate the I/O data for the multiagent system. The desired velocity is given as

$$y_d(t) = 10 \sin \pi t, \quad t \in [0, 1].$$

By considering the sampling time to be $h = 0.001$, we obtain $T = 1000$.

The information flow of the agents is shown in Fig. 13. Select the controller parameter as $\rho = 0.2$, it can be observed that the condition in Theorem 3 is satisfied for all agents. In the simulation, the initial conditions are selected as $x_i^1(1) = x_i^2(1) = 0$, $u_i(1) = 0$, $\hat{\phi}_i(1) = 1$ for all agents.

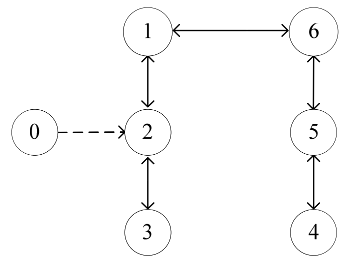

Fig. 13. Communication topology among agents of Example 3.

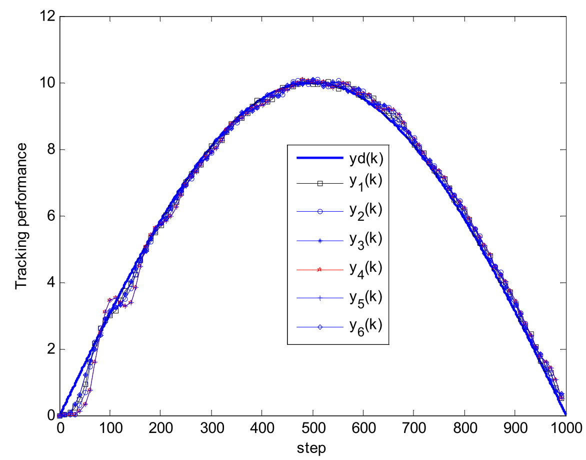

Fig. 14. Tracking performance without measurement noises (Example 3).

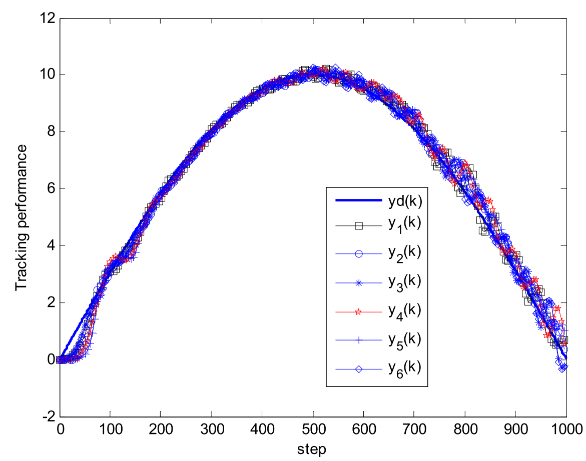

Fig. 15. Tracking performance with measurement noises (Example 3).

The controller parameters are chosen as $\eta = 1$, $\mu = \lambda = 0.5$, $\varepsilon = 10^{-5}$. Fig. 14 gives the system outputs of all agents. We can see that the consensus tracking error can be decreased to vary within a small bound by distributed MFAC. This observation also verifies the consensus tracking result in Theorem 3. Furthermore, we consider that there exists output measurement noise for all agents. The measurement noise is a random signal and its bound belongs to $[-0.05, 0.05]$. From Fig. 15, which shows system outputs of all agents in this case, we can see that the outputs of motors can also follow the desired trajectory with a bound error. However, there exist some deviations in output curves that are caused by stochastic measurement noises. The deviations cannot be canceled since the noise is completely unpredictable.

VI. Conclusion

In this paper, the data-driven consensus tracking control has been considered for multiagent systems with unknown nonlinear dynamics. The dynamical linearization technique has been applied to each agent based on the PPD, and then, a distributed MFAC algorithm has been proposed to ensure that all agents can track the desired trajectory. It can be observe that, for the unknown nonlinear multiagent system, the proposed approach only requires the I/O data of agents rather than the system models. Three examples for multiagent systems have been provided to validate the effectiveness of the proposed method. This paper not only brings novel data-driven design for the problem of consensus but also significantly extend the MFAC approach to multiagent systems. In our future work, we will consider the MFAC consensus problem for MIMO multiagent systems.

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